Effects of Optimal Tactile Feedback in Balancing Tasks: a Pilot Study

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***Abstract*— In this study, we employ optimal control and tactile feedback to teach subjects how to balance a simulated inverted pendulum. The output of a Linear Quadratic Regulator (LQR) was converted to a vibratory teacher-signal and was provided as additional somatosensory feedback to the subjects. The LQR approach is consistent with an energy-saving strategy commonly observed during human motor learning. Our rationale for using the inverted pendulum as a criterion task is that this balance system requires the brain to solve many of the same problems encountered in simple tasks of daily living like transporting a glass of water to the mouth. Experimental results indicate that subjects who trained with the teacher-signal, performed significantly better than subjects who trained only with visual feedback. This result is promising and can be applied, among other fields, in rehabilitation to compensate for lost or compromised proprioception, commonly observed in stroke survivors.**

I. INTRODUCTION

A. *Motivation and Background*

Learning and control of dexterous movements requires timely and reliable sensory feedback [1]. Therefore, it should come as no surprise that tactile and proprioceptive impairments experienced by approximately 50% of stroke survivors negatively impact functional movements and rehabilitation outcomes [2]. Although vision can partly compensate for lost or compromised proprioceptive feedback, delays associated with the visual system result in slow and poorly-coordinated movements [3]. Nevertheless, the primary emphasis of current research and clinical efforts on rehabilitation robotics is directed toward motor retraining ([4], [5]) with only limited focus on enhancing motor learning and re-learning.

The idea of recovering motor skills through synthetic sensory feedback, thus moderating sensory loss, is a reasonable alternative to approaches based only on visual feedback. Audition, electrical stimulation and vibratory signals are all good candidates to assist vision in any task. Considering that electrical stimuli can be painful, these would not make a viable solution in the long run. Moreover, several comparison studies have shown that participants using tactile feedback perform better than those acting on audition [6], [7]. There are also cases in which tactile feedback is at least equally effective as vision, if not more so [7]. For example, in [6] the subjects' performance in one-dimensional pointing tasks was nearly the same regardless of whether they were provided with tactile feedback or visual feedback. In [8] researchers ran a simulator of an automated cockpit system and compared the contribution of different feedback conditions in detecting unexpected status changes. Tactile conditions resulted in higher detection rates for, and faster response times to uncommanded mode transitions. Another pointing study in [9] used a multi-modal mouse to confirm that the cursor was on a target during pointing tasks. Although the overall response times did not vary with feedback type, final positioning time with tactile feedback improved significantly. These findings suggest that tactile feedback could indeed be an effective source of sensory information.

Synthetic tactile feedback as a means to enhance motor (re-)learning and for augmenting somatosensory feedback in general has been explored for many decades [10]. Some successful applications include a wearable robotic bracelet capable of guiding simple movements of the upper limb [11], a tactile vest aimed to support interaction with moving objects at a close range [12], a wearable suit employing error feedback [13] etc. Remarkably, people in [14] report experiencing images in space instead of on the skin when trained with the proposed vibrotactile-visual system. More recently, vibrotactile systems for enhancing postural stabilization in vestibular patients have been proposed and show promise when the synthesized feedback includes all the relevant states [15], [16]. Finally, [17] mentions potential applications of tactile feedback in minimally invasive surgical procedures (MIS), while researchers in [18] describe MusicJacket; a wearable system employing vibrotactile feedback to teach good posture and bowing technique to novice violin players.

In summary, although there is promising work on the exploitation of vibrotactile stimulation as a means of augmenting sensory feedback, there is still much to be done before it may become possible to develop vibrotactile stimulation systems effective in promoting recovery of sensory-motor skills. Moreover, the literature provides no strong theoretical and computational basis for designing synthetic sensory feedback for use in promoting motor learning and relearning within the context of goal-directed limb movements.

B. *Objective*

In this experiment, we employ optimal control and tactile feedback to teach unimpaired subjects how to balance a simulated inverted pendulum. Unlike prior studies, the re-

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sulting interface encodes an optimized linear combination of state information, thus always suggesting the optimal course of action. In particular, the output of a Linear Quadratic Regulator (LQR) is converted to a vibratory teacher-signal and is provided as additional somatosensory feedback to the subjects, thus introducing the human factor in the control loop. The LQR output is no longer fed back to the system, so it is then up to the subjects to decide how to integrate this signal in their attempt to balance the pendulum. Our rationale for using the well-known cart-and-pole task is that this balance system requires the brain to solve many of the same problems necessary for simple everyday tasks like transporting a glass of water or a spoonful of peas to the mouth.

The current study falls under the broader category of Human Machine Interface (HMI) and offers a demonstration of how human performance can be improved by using optimal controllers to generate artificial feedback to a human operator. We sought to determine whether subjects attempt to learn the optimal behavior conveyed by the LQR, i.e. whether they try to keep the energy of the closed-loop system as small as possible. Such a strategy would be consistent with the energy-saving approach commonly observed in human behavior.

II. Materials and Methods

A. Overview

Fig. 1 shows a block diagram of the experimental setup. Our goal was to easily compare and combine visual and tactile feedback in terms of motor learning and to examine whether complex information can be successfully encoded and shared by small vibrotactile motors (tactors). In this experiment, all hypotheses were tested on a simulated two-dimensional inverted pendulum. During simulation, subjects could directly control the cart position by moving their right hand along a horizontal axis (a one-dimensional task). The major components of the testing platform are described below:

1) Robot Operating System (ROS): ROS [19] is a distributed framework of processes (called nodes) that communicate via message passing. It provides all the standard services of a typical operating system and handles the integration of all hardware and software parts of the experiment.

2) Microsoft Kinect: The Kinect uses an infra-red structured light array to build a three-dimensional point cloud representation of its view of the world. ROS was used to collect and process the data coming in from the Kinect at approximately 30Hz. Interface to the Kinect is provided by the NITE™ open source Kinect drivers released by PrimeSense™ [20].

3) `trep`: We have developed a simulation package called `trep` (available at https://code.google.com/p/trep/) which allows for dynamic simulation of arbitrary mechanical systems in generalized coordinates based on the variational integrator approach [21]. This is where the dynamics of the pendulum/cart system were simulated. ROS was responsible for mapping the subject's right hand position, as captured by the Kinect, to the corresponding cart position along a horizontal axis.

4) Tactors/Arduino: Tactile feedback was provided by vibrating motors typical of those found in cellphones. Their compact size (5mm radius) and high output to power ratio are ideal for our setup. The tactors are controlled by an Arduino microcontroller board (through ROS) using PWM signals.

B. LQR Loop

The purpose of this study was to teach subjects how to successfully balance an inverted pendulum. We hypothesized that the most fitting way to do that would be to first design an ideal controller for the system and then train subjects to take advantage of that controller's output.

The nonlinear dynamics of the simulated pendulum shown in Fig. 2a are given by:

$$(M+m)\ddot{x}_c + ml\dot{\theta}^2 \sin\theta - ml\ddot{\theta}\cos\theta = F \tag{1}$$

$$l\ddot{\theta} - g\sin\theta = \ddot{x}_c \cos\theta \tag{2}$$

where all symbols are explained in Table I. Linearization about the vertically upward equilibrium position, $\theta = 0$, gives the following state-space equations:

$$\begin{bmatrix} \dot{x}_c \\ \ddot{x}_c \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{mg}{M} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{(M+m)gl}{Ml^2} & 0 \end{bmatrix} \begin{bmatrix} x_c \\ \dot{x}_c \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ \frac{1}{Ml} \end{bmatrix} u \tag{3}$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_c \\ \dot{x}_c \\ \theta \\ \dot{\theta} \end{bmatrix} \tag{4}$$

where u has been substituted for the input F. A LQR loop as in Fig. 2b can now be designed to meet the desired response. The optimal weighting of states was computed off-line while the output of the LQR was computed in real-time as described in the following section.

C. Using the LQR as a "teacher"

The derived controller carried all the necessary state information needed to balance the pendulum at a reference cart position r. We used two tactors in this study to map the signed control input u to each tactor's voltage input. This was

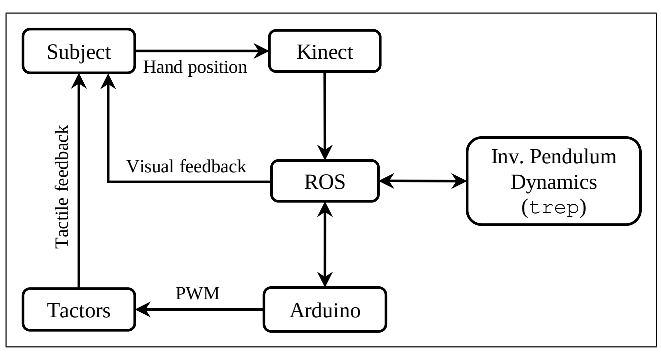

Fig. 1. System overview. Arrows indicate flow of information.

accomplished by passing u through a linear bounded function as demonstrated in Fig. 2c and Fig. 2d. Thus, both the target position and direction of movement could be encoded by regulating the amplitude of vibration of the appropriate tactor. Theoretically, if someone could “follow” accurately the resulting vibrotactile feedback v they should be able to successfully complete the balancing task.

Note that even though u was by itself capable of balancing the pendulum, it was only used to synthesize the “teacher”; it was no longer used as an input to the simulated cart-pole system. Instead, the input signal was based on the subject’s right hand position, captured by the Kinect. ROS matches these hand data to the corresponding cart position, thus creating the actual input signal h (Fig. 2c). As a result, once the subject received the “suggested” feedback v, it was entirely up to them to decide how they should move their hand. This idea extends the conventional implementation of a HMI where the subjects seek to improve their performance by utilizing external feedback signals. What is unique in the current implementation is that subjects were expected to learn the optimal, energy-minimizing behavior instructed by the LQR that would otherwise be unintuitive and which would require considerable exploration to obtain.

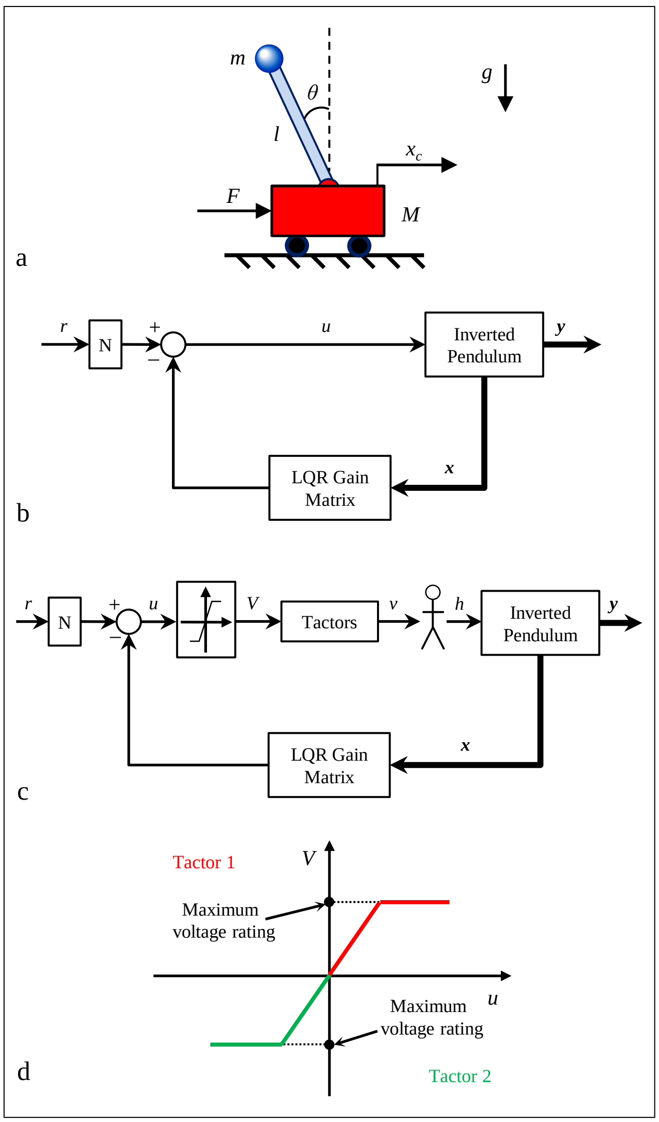

Fig. 2. (a) A schematic of the inverted pendulum/cart system. The rod is considered massless. (b) A typical LQR control loop. (c) Extended LQR loop where the controller is used to ”teach” the user how to balance the system. (d) Mapping from u to V. All symbols are explained in Table I.

D. Experimental Protocol

Ten subjects (4 females, 6 males) consented to participate in this pilot study, which was approved by Northwestern University’s Institutional Review Board. Only adult subjects with no history of neurological disorders, and no prior knowledge of the experimental procedure were allowed to participate.

We sought to determine whether the proposed type of tactile feedback can improve learning of a new motor skill. Subjects were divided into two groups of five and were asked to play a “video game” for approximately 20 minutes. The game had two phases, a training phase and a testing phase (10 minutes each with a 5-minute break in-between). The goal was to keep the pendulum from falling for as long as possible. If the pendulum fell, a new session started immediately. Trials continued until the end of the experiment. We found that 10 minutes of training sufficed to learn the task without noticeable fatigue effects. During the training phase, one group received both visual and tactile feedback (TV group) while the other group received only visual feedback (V group). During the testing phase, both groups received only visual feedback of the pendulum/cart system (i.e. no vibrotactile feedback).

The experimental setup is shown in Fig. 3. In a typical session, subjects stood in front of a monitor and used their right hand to control the position of the cart. In the training phase, the TV group also received tactile feedback by two small tactors as shown in Fig. 3. To acquire experience with the artificial feedback, i.e. what the amplitude of the vibration means and how they should use it, the TV group initially practiced a target-matching task for 1-2 minutes. In that familiarization task, subjects were asked to match a moving cursor (representing their right hand) to a randomly changing target along a horizontal axis. During the task, error feedback

TABLE I

SYMBOL DESCRIPTIONS

Fig. 2a		Fig. 2b-c	
Symbol	Description	Symbol	Description
M	Mass of the cart	r	Reference cart position
m	Point mass	u	Control signal
l	Pendulum length	x	State vector $[x_c\ \dot{x}_c\ \theta\ \dot{\theta}]^T$
F	External force	y	Output vector $[x_c\ \theta]^T$
x_c	Cart position	N	Precompensator
θ	Pendulum angle	V	Voltage
g	Gravity	v	Vibrotactile feedback
		h	Hand position

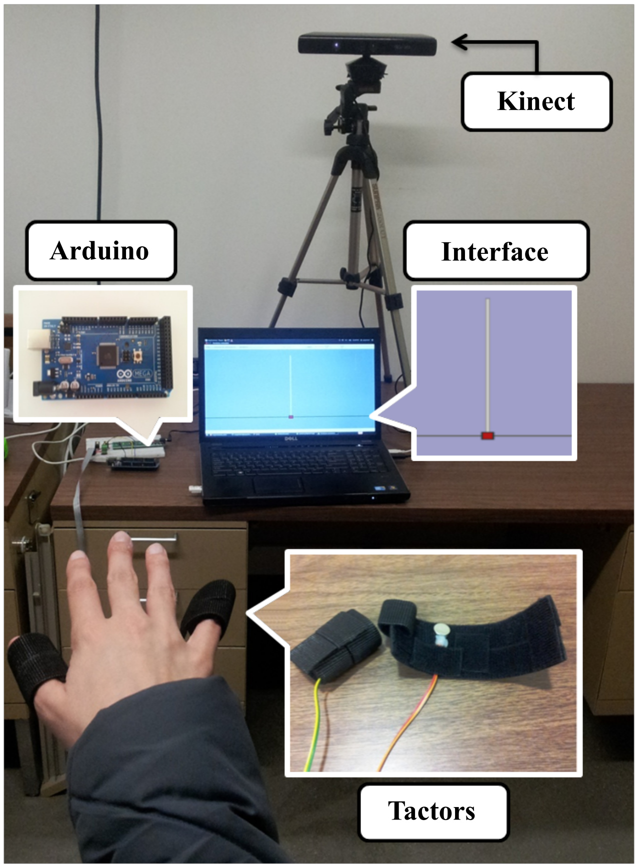

Fig. 3. Experimental setup.

was provided through the tactors, thus familiarizing the group with vibrotactile feedback. This type of feedback provided in this familiarization task was entirely different from that provided in the primary task and therefore their performance on the latter was not affected.

III. Results

A. Preliminary Analysis

We used the Time To Failure (TTF) as our primary performance metric, which we computed for each trial. TTF was the amount of time (in seconds) during which the subject was able to keep the pendulum from falling. Fig. 4 illustrates a typical response of a subject in the TV group during the training phase. For this trial, the subject clearly attempted to "follow" the vibrotactile feedback conveying the optimal response. By doing so, the subject was able to balance the inverted pendulum for more than 60 seconds (more than 4 times the average V group performance).

Fig. 5a (top) shows the within-group average of the first 13 trials of the training and testing phases (i.e. the Mean Time To Failure, MTTF). This preliminary comparison suggests that after an initial startup transient in the training phase, the TV group performed better than the V group in almost every trial. Thus, an optimal vibrotactile training signal appears to provide information that people can use to improve performance of a complex motor task, above and beyond that which is capable using only visual feedback of manipulated object state. Even so, one can notice that there is substantial variability between trials in the TV group. This variability continues throughout the whole training phase suggesting that people may require more than 10 minutes of training to fully learn how to capitalize on the training signal. Future studies will investigate the patterns of exploration and the time course of variability reduction as people learn to use optimal vibrotactile feedback to perform this challenging balancing task.

Fig. 5a (bottom) reveals a carry-over benefit of TV training into the testing phase over and above what was acquired in the vision only testing condition. This important result demonstrated that the skill learned during the short training period generalized to the testing condition without vibrotactile feedback. That is, by attempting to follow a training signal that combines controlled-object states in a way we deemed to be optimal, subjects may have implicitly learned how to extract and combine those states all without relying on the training signal. Future studies will investigate how to maximize the performance gains obtainable via augmented LQR state feedback and how to maximize generalization to situations wherein training signals are no longer available.

B. Statistical Results

To quantify the carry-over effect described above, we fit the individual performance curves to the TTF data obtained from each subject:

$$\alpha(1 - e^{-\frac{t}{\beta}}) + \gamma \tag{5}$$

where t is trial number, α is the gain (i.e. the amount learned or the steady-state performance), β is the time constant (i.e. the learning rate) and γ is an offset. The resulting approximations were reasonably similar to the actual performance curves ($r^2 > 0.75$).

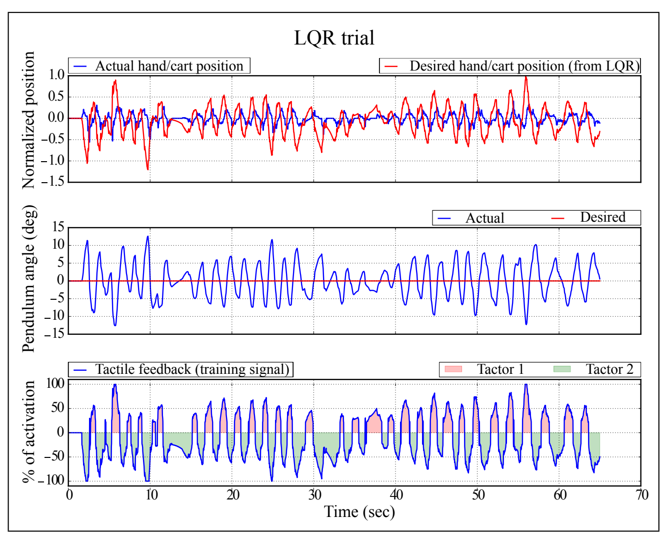

Fig. 4. Typical response of a subject training with both tactile and visual feedback. The percentage of activation is equivalent to the amplitude of vibration.

Three two-sample t-tests were then performed comparing each model parameter across groups in the testing phase. Subjects in the TV group had significantly better steady-state behavior (referring to $\alpha + \gamma$) than the V group (*mean* = 26.5, *SD* = 1.81 as opposed to *mean* = 19.07, *SD* = 1.95), $t(8) = 6.25$, $p < 0.05$. This is consistent with the carry-over effect that was observed in Fig. 5a. In contrast, no significant difference was found between the time constants of the TV group (*mean* = 1.908, *SD* = 0.169) and the V group (*mean* = 1.973, *SD* = 0.65), $t(8) = -0.22$, $p = 0.833$. Likewise the offset of the TV group (*mean* = −8.15, *SD* = 1.71) was not different from the V group (*mean* = −5.35, *SD* = 2.97), $t(8) = -1.83$, $p = 0.105$. Using (5) we additionally approximated the average performance curve of each group (Fig. 5b). These curves graphically support the individual-subjects results.

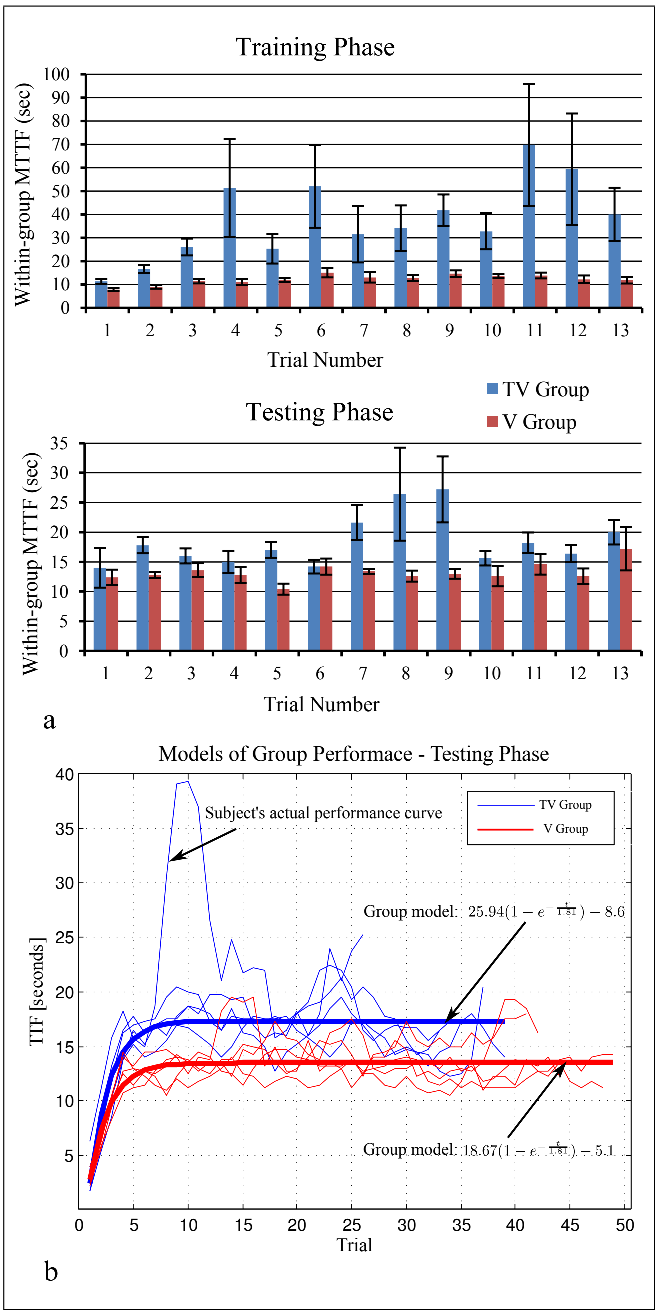

Fig. 5. (a) First 13 trials of the experiment. Error bars show standard error. (b) Approximated model of group performance in the testing phase.

The same conclusion about the steady-state performance can be reached by directly comparing the raw TTF data across groups. We used two two-sample t-tests to compare the group averages of the first two trials (TTFs) at the beginning of testing and the average of the last 10 trials at the end of testing. The results show that the TV group (*mean* = 17.2, *SD* = 2.06) outperformed the V group (*mean* = 14.22, *SD* = 1.81) at the end of testing phase ($t(7) = 2.43$, $p < 0.05$) despite no significant difference between groups at the start of testing (*mean* = 15.9, *SD* = 4.38 for the TV group and *mean* = 12.6, *SD* = 0.962 for the V group, $t(4) = 1.65$, $p = 0.175$). The number of trials at the beginning is limited to two because of the steep learning curve at the beginning of the testing phase (Fig. 5b). This steep transient response is characterized by a time constant of approximately 2 trials and is likely due to the 5-minute intermediate break. The fact that the time constants of the two groups in the testing phase do not significantly differ, implies a smooth transition of the TV group to the new feedback condition.

Finally, we examined the steady-state performance in the training phase to confirm that tactile feedback is actually useful as implied by Fig. 5a. Indeed, after comparing the average of the last 5 trials for each subject, the TV group (*mean* = 46.22, *SD* = 22.59) was significantly better than the V group (*mean* = 13.52, *SD* = 1.17), $t(4.021) = 3.232$, $p = 0.03$.

IV. Discussion

In this study we used a vibrotactile feedback system and LQR design techniques to test the utility of synthetic state feedback to facilitate learning of a challenging object control task that emulates many tasks of daily living. We found that by providing vibrotactile feedback of object states in the specific combination that leads to optimal performance, subjects enjoyed an immediate enhancement of task performance over those provided only ongoing visual feedback of object motion. This effect was evident within the first minute or so of training and persisted over the entire training period (10 minutes). Importantly, we observed that this training benefit generalized to the testing interval wherein subjects received only visual feedback of object motion. This implies that training to emulate a response driven by LQR optimal feedback can induce subjects to learn which combination of object states can drive motor behavior so as to achieve enhanced performance of a challenging object manipulation task. The results suggest that vibrotactile feedback of synthetic state information may be an effective tool for use in optimizing motor performance in sports and, perhaps, for optimizing motor re-learning following neuromotor injury (e.g. stroke).

The most novel feature of our study is the use of a LQR training signal to guide task performance. The LQR training signal carried information about all task-relevant state variables in exactly the proper proportion required to optimize performance. To our knowledge, all prior studies of vibrotactile display systems have used tactors to provide feedback of just one or two state variable such as position, velocity or error information. And while it is theoretically possible that people could have derived the relevant cart-and-pole state feedback from vision (and subsequently learn to combine those states in an optimal manner), the observation of an immediate training benefit of LQR training demonstrates that they do not do so - at least within the time frame we examined. The observation that beneficial effects of LQR vibrotactile feedback training transfer to trials without such feedback demonstrates that the brain *can* extract the task-relevant object state feedback from visual feedback of object motion under the proper training conditions. Future studies should determine the frequency, duration, and optimal scheduling protocols for LQR vibrotactile feedback training that seeks to promote optimization of task performance, its generalizability to other tasks of daily living, and the extent to which performance enhancements can be retained over time.

V. Conclusions

In this pilot study, we presented a new way to synthesize artificial sensory feedback for the express purpose of enhancing the ability to learn new motor skills. We combined an optimal controller with vibrotactile feedback, and the experimental results showed that subjects who trained with the LQR teacher signal performed better than those who trained only with visual feedback of object motion. Potential applications of this new sensory feedback enhancement technique include cases of visual impairment, performance optimization in sports, skill optimization in the teleoperation of surgical tools and in the physical rehabilitation of limb movements following neuromotor injury, especially in those cases where limb proprioception is compromised.

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References

[1] F. A. Mussa-Ivaldi and L. E. Miller, "Brain-machine interfaces: computational demands and clinical needs meet basic neuroscience," *Trends in Neurosciences*, vol. 26, no. 6, pp. 329–334, 2003.

[2] L. M. Carey, "Somatosensory Loss after Stroke," *Critical Reviews in Physical and Rehabilitation Medicine*, vol. 7, no. 1, pp. 51–91, 1995.

[3] R. L. Sainburg, H. Poizner, and C. Ghez, "Loss of Proprioception Produces Deficits in Interjoint Coordination," *Journal of Neurophysiology*, vol. 70, no. 5, pp. 2136–2147, 1993.

[4] S. E. Fasoli, H. I. Krebs, J. Stein, W. R. Frontera, and N. Hogan, "Effects of Robotic Therapy on Motor Impairment and Recovery in Chronic Stroke," *Archives of Physical Medicine and Rehabilitation*, vol. 84, no. 4, pp. 477–482, 2003.

[5] D. J. Reinkensmeyer and J. L. Patton, "Can Robots Help the Learning of Skilled Actions?" *Exercise and Sport Sciences Reviews*, vol. 37, no. 1, pp. 43–51, 2009.

[6] E. Charoenchaimonkon, P. Janecek, M. N. Dailey, and A. Suchato, "A Comparison of Audio and Tactile Displays for Non-Visual Target Selection Tasks," in *Proc. IEEE International Conference on User Science and Engineering (i-USEr '10)*, 2010, pp. 238–243.

[7] M. Sun, X. Ren, and X. Cao, "Effects of Multimodal Error Feedback on Human Performance in Steering Tasks," *Journal of Information Processing*, vol. 18, no. 0, pp. 284–292, 2010.

[8] A. E. Sklar and N. B. Sarter, "Good Vibrations: Tactile Feedback in Support of Attention Allocation and Human-Automation Coordination in Event- Driven Domains," *Human Factors: The Journal of the Human Factors and Ergonomics Society*, vol. 41, no. 4, pp. 543–552, 1999.

[9] M. Akamatsu, I. S. MacKenzie, and T. Hasbrouc, "A Comparison of Tactile, Auditory, and Visual Feedback in a Pointing Task Using a Mouse-Type Device," *Ergonomics*, vol. 38, no. 4, pp. 816–827, 1995.

[10] B. W. White, F. A. Saunders, L. Scadden, P. Bach-Y-Rita, and C. C. Collins, "Seeing with the skin," *Perception and Psychophysics*, vol. 7, no. 1, pp. 23–27, 1970.

[11] F. Sergi, D. Accoto, D. Campolo, and E. Guglielmelli, "Forearm Orientation Guidance with a Vibrotactile Feedback Bracelet: on the Directionality of Tactile Motor Communication," in *Proc. IEEE/RAS-EMBS International Conference on Biomedical Robotics and Biomechatronics*, Scottsdale, USA, Oct. 2008, pp. 433–438.

[12] U. Yang, Y. Jang, and G. J. Kim, "Designing a Vibro-Tactile Wear for "Close Range" Interaction for VR-based Motion Training," in *Proc. International Conference on Artificial Reality and Telexistence*, 2002, pp. 4–9.

[13] J. Lieberman and C. Breazeal, "TIKL: Development of a Wearable Vibrotactile Feedback Suit for Improved Human Motor Learning," *IEEE Transactions on Robotics*, vol. 23, no. 5, pp. 919–926, 2007.

[14] P. B. y Rita and S. W. Kercel, "Sensory substitution and the human machine interface," *Trends in Cognitive Sciences*, vol. 7, no. 12, pp. 541–546, 2003.

[15] B.-C. Lee, J. Kim, S. Chen, and K. H. Sienko, "Cell phone based balance trainer," *Journal of NeuroEngineering and Rehabilitation*, vol. 9, no. 10, 2012.

[16] B.-C. Lee, S. Chen, and K. H. Sienko, "A Wearable Device for Real-Time Motion Error Detection and Vibrotactile Instructional Cuing," *IEEE Trans. Neural Syst. Rehab. Eng.*, vol. 19, no. 4, pp. 374–381, 2011.

[17] P. Puangmali, K. Althoefer, L. D. Seneviratne, D. Murphy, and P. Dasgupta, "State-of-the-Art in Force and Tactile Sensing for Minimally Invasive Surgery," *IEEE Sensors J.*, vol. 8, no. 4, pp. 371–381, 2008.

[18] J. van der Linden, E. Schoonderwaldt, J. Bird, and R. Johnson, "MusicJacket-Combining Motion Capture and Vibrotactile Feedback to Teach Violin Bowing," *IEEE Trans. Instrum. Meas.*, vol. 60, no. 1, pp. 104–113, 2011.

[19] (2013) Robot Operating System. Willow Garage. [Online]. Available: http://www.ros.org/

[20] (2013) PrimeSense NITE Middleware. PrimeSense. [Online]. Available: http://www.primesense.com/

[21] E. R. Johnson and T. D. Murphey, "Scalable Variational Integrators for Constrained Mechanical Systems in Generalized Coordinates," *IEEE Transactions on Robotics*, vol. 25, no. 6, pp. 1249–1261, 2009.